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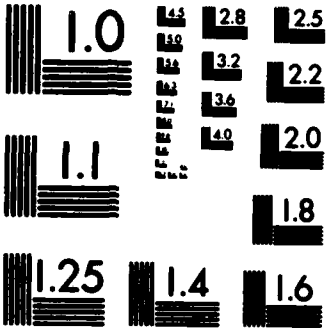

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20. ABSTRACT

A detection algorithm for a new class of functions ~~(defined in the paper)~~ has been presented. This algorithm not only detects such functions but also provides the decomposition to facilitate automatic layout and helps in finding a faster circuit and an efficient layout. If an example (two inputs) is given to verify that the function is not symmetric then this detection algorithms asks a polynomial (on number of variables) number of queries about the function and comes up with a circuit if one exists.

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Introduction:

In VLSI, it is well known that a faster circuit and an efficient layout are strongly dependent on the nature of the function to be implemented. It has been found that a reasonably faster circuit with an efficient layout can be achieved if the function is symmetric. Unfortunately many functions occurring in a problem may not fall into the category of symmetric function. So it is natural to consider the layout complexity of *somewhat symmetric* function. In this report we have introduced a notion of *bi symmetric* function. As far as faster circuit and efficient layout are concerned this class of function has almost all the nice properties of a symmetric function. An algorithm has been developed to check whether a given function is bi symmetric or not. With some help this algorithm asks polynomially (on number of variables) many queries only.

Definition 1: A function $F: (0,1)^n \rightarrow (0,1)$ is bi symmetric if there exists these three functions F', G_1 and G_2 such that $F(x_1,x_2,\cdots,x_n) = F'(G_1(x_{j_1},x_{j_2},\cdots,x_{j_i}), G_2(x_{j_{i+1}},x_{j_{i+2}},\cdots,x_{j_n}))$, where the set $\{x_1,x_2,\cdots,x_n\} = \{x_{j_1},x_{j_2},\cdots,x_{j_n}\}$ and the functions G_1 and G_2 are symmetric. Figure 1 clearly explains the definition in the circuit format.

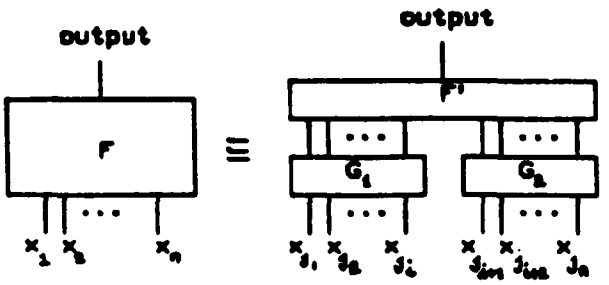

Figure 1.

Since the functions G_1 and G_2 are symmetric, they can be realized by a reasonably fast circuit and an efficient layout. Maximum number of output bits for both G_1 and G_2 is of the order of $log(n)$. Hence the function F' is comparatively simple to implement.

Given a function F, known to be not symmetric, we would like to know whether it is bi symmetric or not. An algorithm has been developed which asks queries on the given function and outputs the three functions F', G_1 and G_2 such that latter two are symmetric (*fig.* 2). They satisfy the property that $F = F'$ if and only if F is bi symmetric.

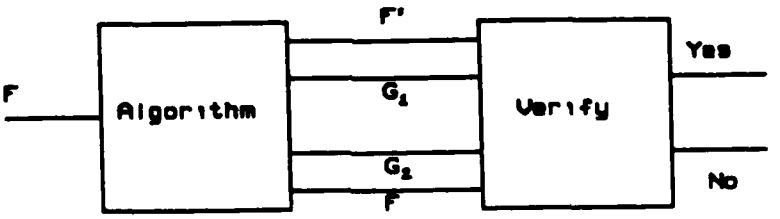

Figure 2.

Definition 2: 2^n combinations of n bit input can be grouped into $(n+1)$ different categories called Level(0), Level(1), ... , Level(n) such that all strings with exactly k ones are in Level(k) and not in any other.

Lemma 1: If F is not symmetric then there exists a k, x and y such that $x, y \in$ Level(k) and $F(x) \neq F(y)$.

Proof (by contradiction): If there exists no k such that $x, y \in$ Level(k) and $F(x) \neq F(y)$ then for all k and for all pairs $x, y \in$ Level(k) $F(x) = F(y)$. So F is symmetric (a contradiction).□

Lemma 2: If F is not symmetric then there exists a k, x and y such that $x, y \in \text{Level}(k)$ and $F(x) \neq F(y)$ and hamming distance between x and y is two (i.e., positions of a zero and one are swapped).

Proof (by construction): From Lemma 1, we know that there exists a k, x and y such that $x, y \in \text{Level}(k)$ and $F(x) \neq F(y)$. Since x,y$\in$*Level*(k) the number of zero positions in x that are one's in y is equal to the number of one positions in x that are zero's in y. Let that number be l ($\leq k$). Now one can march from x towards y, each time correcting one pair. It can be represented by the following sequence:

$$x\ (=z_0), z_1, \cdots, z_{l-1}, y\ (=z_l)$$

where z_i is obtained by swapping the positions of a zero and one from z_{i-1} and it is closer to y (hamming distance measure). Note that hamming distance between z_i and z_{i+1} is two. Since $F(x) \neq F(y)$ there must be an i such that $F(z_i) \neq F(z_{i+1})$.□

Fact 1: If F is a bi symmetric and there exists an x,y$\in$*Level*(k) such that hamming distance between x and y is two and $F(x) \neq F(y)$, then those bits they differ from one another must be in different G_i (i=1,2).

Proof (by contradiction): Without loss of generality, assume that both bits belong to G_1. Since G_1 is symmetric, swapping two bits should not result in different output (a contradiction).□

Lemma 3: Let F be a bi symmetric function and there exists an x,y$\in$*Level*(k) such that hamming distance between x and y is two and $F(x) \neq F(y)$. Without loss of generality assume that b_i and b_j are the two bits differing (causing the hamming distance) in x and y and the former is an input to G_1 and latter to G_2. Let b_i th bit in x be one. The following statements are true.

1). All zero's positions in x, that change output when swapped with the b_i th, belong to G_2 and all other zero's positions belong to G_1.

2). All one's positions in x, that change output when swapped with the b_j th, belong to G_1 and all other one's positions belong to G_2.

Proof: Since Gs are symmetric functions we can just concentrate on number of ones in the input. We observe that if we lose one 1 from G_1 and gain one 1 in G_2 then the result change from the original (original input being x). So the same should happen if we swap a one (b_i) to any one of the zeros in G_2. So all zeros in x that are input to G_2 must change the result when swapped with b_i. One can make a similar argument to second statement also.□

Algorithm:

This algorithm consists of three stages. First the input is partitioned into two parts where one is input to G_1 and the other to G_2. In the second stage of the algorithm, the truth table of the three functions F' ,G_1 and G_2 are obtained. The third stage just verifies whether the resultant composition is F or not.

Stage 1 (partitioning the given input into two parts):

1. Assume that the given function is bi symmetric.
2. Find a string x such that there exists a y with the property x,y$\in$*Level*(k), hamming distance between x and y is two and $F(x) \neq F(y)$. Now apply the procedure described in Lemma 3 to find the partition.

For ease of presentation we are assuming that the number of inputs to G_1 and G_2 are equal. Let it be m. Since Gs are symmetric functions, we can pick representative for each Level(0), Level(1), ... ,Level(m). Now form a boolean matrix M (($m+1$) by ($m+1$)) which is the result of F for various pairs of these ($m+1$) , ($m+1$) representative inputs for each G_1 and G_2. If F is a bi symmetric function, this matrix can be rewritten by permuting some rows and columns such that we get monochromatic rectangles with the nice pattern shown in fig. 3. Note that the value written inside the rectangles in fig. 3 is arbitrary.

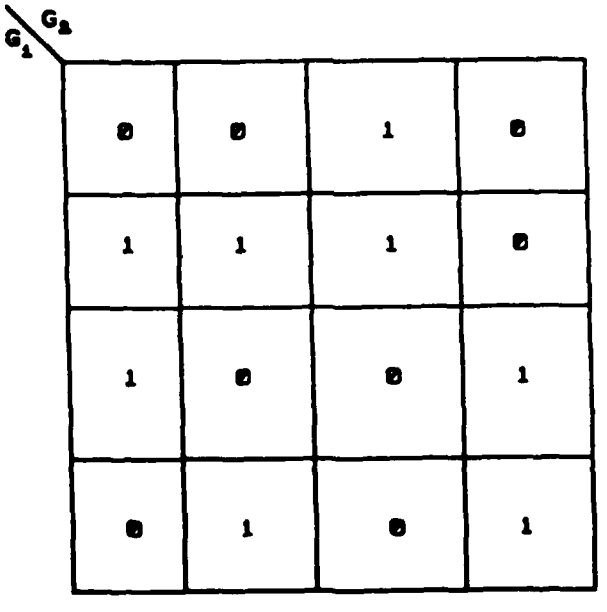

Figure 3.

At least there exists a trivial partition such that each monochromatic rectangle has only one element.

Stage 2: (Finding the three functions)

1. Pick those representatives and form the matrix M.
2. Collect all identical rows and bunch them together (kind of sorting).
3. Now collect all identical columns and bunch them together (kind of sorting).

Step two and three creates monochromatic rectangles. Let the rows be divided into k_r parts and the columns k_c parts. Let the row side be G_1 and column side be G_2. Now the number of output bits for G_1 is $\lceil log(k_r) \rceil$. One can easily write coding for each of the k_r parts using $\lceil log(k_r) \rceil$

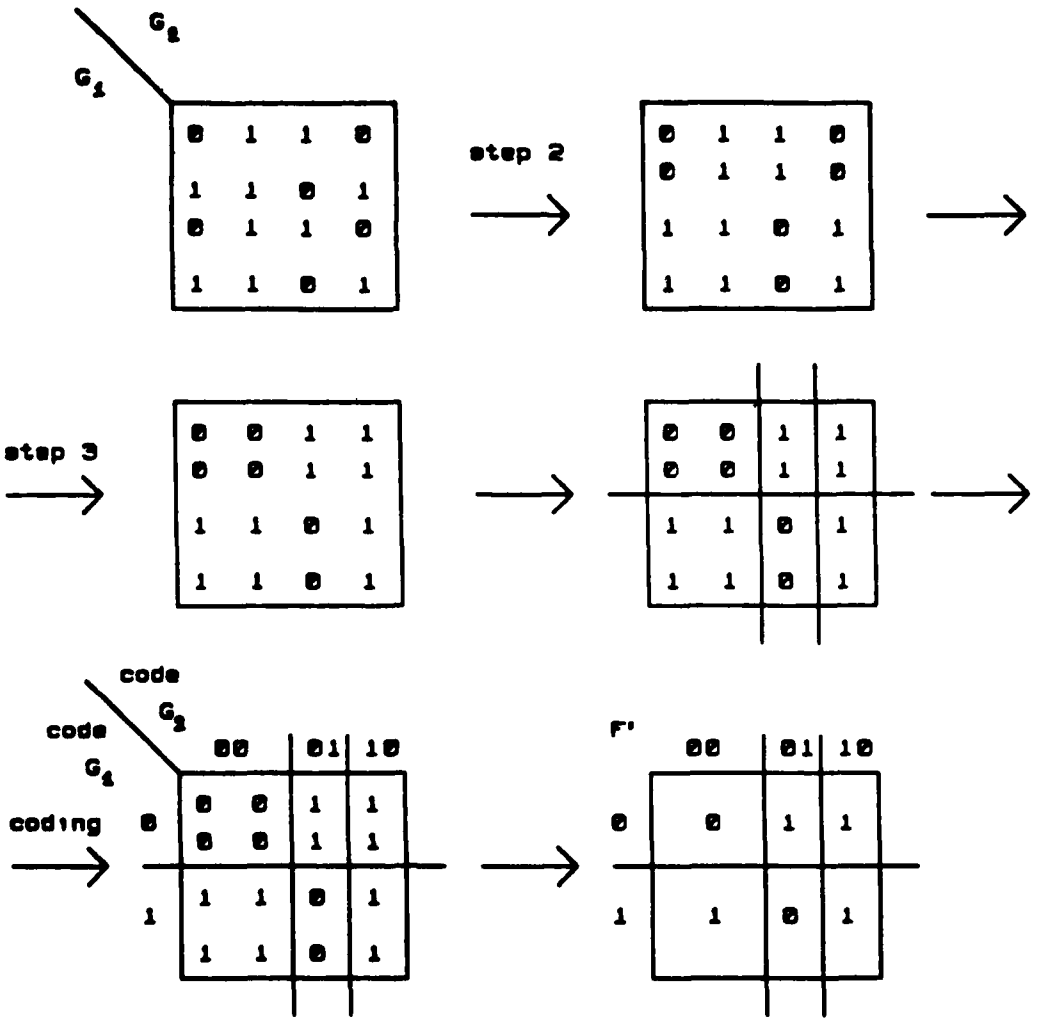

Figure 4.

bits. So the G_1 function is nothing but the coder. Similarly one can also find G_2. Now the truth table for F' is derived by replacing each part in the row by its code and each part in the column by its code and the monochromatic rectangles are replaced by the single value it has. Figure 4. illustrates the algorithm clearly.

Complexity Analysis and Conclusion:

The first part of the second step in the stage 1 of the algorithm is the help needed to run the algorithm in polynomial number of queries. Otherwise the number of queries (complexity) of stage 1 is order n. The second stage takes only order m^2 steps where $m \leq n$. The third stage of the algorithm is to verify whether $F = F'$. This will be a thorough verification and it will take queries as many as the size of the truth table of F. If we believe that the given function is bi symmetric the verification is not needed.

It should be interesting to know if there exists an algorithm for somewhat symmetric functions with three or more G's.

END

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